

Research on Position and Posture Trajectory Planning of 6R Industrial Robot

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Keywords: Industrial robot; Kinematics analysis; Trajectory planning; Simulation

Abstract: Taking Fanuc Mate 200iD-4S six-degree-of-freedom robot as the research object, the three-dimensional model of the robot is established by SolidWorks software, and the joint coordinate system and the bar parameters are determined by D-H method. The kinematics model of the corresponding robot is constructed. Using the Robotics Toolbox module of Matlab software, the forward motion and the inverse motion of the robot are solved respectively. The trajectory planning of joint space and Cartesian space is constructed respectively. The angular position, angular velocity and angular acceleration of each joint are analyzed in detail with time. The rationality of the structural parameter design of the robot is verified.

1. Introduction

Starting from the parameter design of Fanuc Mate 200iD-4S industrial robot, the simulation model of the robot is established. The structure and D-H parameters of the robot are analyzed by using the theory of homogeneous coordinate transformation^[1]. The trajectory of the robot is planned in the joint space and Cartesian space respectively, and its motion characteristics are analyzed^[2]. It can not only directly reflect the end-effector in the basic coordinate system. The position and posture of the robot vary with the joint variables, and can reflect the relationship between the joint velocity and the terminal operating speed of the robot. At the same time, the relationship between the position, velocity and acceleration of the joints and the time is obtained. During the simulation process, the motion of the robot can be observed, and the results can be visually displayed in the form of graphics^[3].

2. Establishment of D-H Model of Fanuc Mate 200iD-4S Robot

The D-H method is used to establish the coordinate system of connecting rods at each joint of Fanuc Mate 200iD-4S industrial robot, in order to describe the motion relationship between adjacent two rods and the position and posture of the end. Denavit and Hartenberg published an article in the Journal of Applied Mechanics of the American Association of Mechanical Engineers in 1955. In this paper, a matrix method for establishing the coordinate system of rods is proposed, which is called D-H method^[4]. Because this method is simple and easy to understand, and is not affected by the configuration and complexity of the robot, it can be used to represent the transformation in any coordinate system, so it is recognized as a standard method to represent the robot and to model the motion of the robot. In this method, the matrix relation of homogeneous transformation is used to obtain the position and posture of the end effector relative to the base coordinate system, and then the kinematics equation of the industrial robot is established^[5].

According to the actual size and six-axis rotation characteristics of Fanuc Mate 200iD-4S robot, the coordinate system model of each joint connecting rod is constructed, as shown in Fig.1. At the same time, the construction rules of D-H connecting rod coordinate system are determined.^[6]:

- 1) Origin o_i : At the common intersection of the connecting rod length a_i and z_{i+1} axis
- 2) Axis x_i : In line with the common normal of axis z_i and z_{i-1} , the direction of coordinate

axis is far from the direction of axis z_{i-1} ;

- 3) Axis y_i : According to the determined x_i and z_i axes, according to the right-hand rule;
- 4) Axis z_i : coincide with the axis of joint z_{i+1} , the direction of coordinate axis is arbitrary.

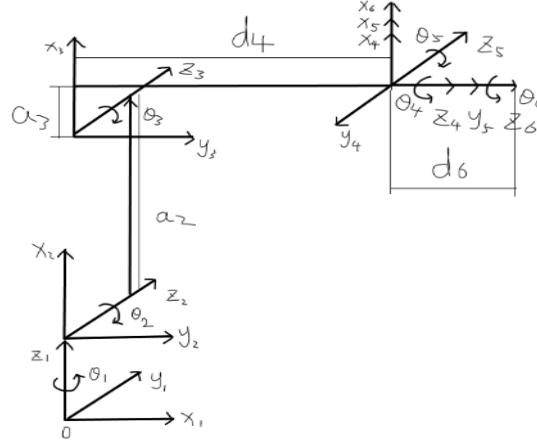


Figure 1. D-H linkage coordinate system of Fanuc Mate 200iD-4S robot

Based on the model of joint linkage coordinate system, the D-H parameters of Fanuc Mate 200iD-4S robot are determined according to the construction rules of D-H linkage coordinate system and the relationship between the links, as shown in Table 1.

Table 1 D-H parameters of Fanuc Mate 200iD-4S robot link

joint i	Twist angle α_{i-1}	Setover d_i	Rod length a_{i-1}	angle θ_i	Variable range
1	0°	0	0	θ_1	$(\pm 170^\circ)$
2	-90°	0	0	θ_2	$(-30^\circ \sim 150^\circ)$
3	0°	0	a_2	θ_3	$(-40^\circ \sim 120^\circ)$
4	-90°	d_4	a_3	θ_4	$(\pm 180^\circ)$
5	90°	0	0	θ_5	$(\pm 140^\circ)$
6	-90°	d_6	0	θ_6	$(\pm 360^\circ)$

Among them, θ_i is the joint angle of i , a_i is the rod length between adjacent joints, and d_i is the rod offset between adjacent joints. α_i is the twist angle between adjacent coordinate systems, Among them, $a_2 = 260$ mm, $a_3 = 20$ mm, $d_4 = 290$ mm, $d_6 = 70$ mm, α 、 a 、 d is a fixed value and θ is a variable.

The general formula of transformation matrix of adjacent link coordinate system of Fanuc Mate 200iD-4S robot is ^[7]:

$${}^{i-1}T_i = \begin{bmatrix} c\theta_i & -s\theta_i & 0 & a_{i-1} \\ s\theta_i c\alpha_{i-1} & c\theta_i c\alpha_{i-1} & -s\alpha_{i-1} & -d_i s\alpha_{i-1} \\ s\theta_i s\alpha_{i-1} & c\theta_i s\alpha_{i-1} & c\alpha_{i-1} & d_i c\alpha_{i-1} \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (1)$$

Eq. 1: $c\theta_i$ denotes $\cos(\theta_i)$, $s\theta_i$ denotes $\sin(\theta_i)$.

By substituting the parameters in Table 1 into Eq. 1, the transformation matrix of adjacent connecting rods is obtained as follows:

$$\begin{aligned}
{}^0_1T &= \begin{bmatrix} c\theta_1 & -s\theta_1 & 0 & 0 \\ -s\theta_1 & -c\theta_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} {}^1_2T = \begin{bmatrix} c\theta_2 & -s\theta_2 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ -s\theta_2 & -c\theta_2 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} {}^2_3T = \begin{bmatrix} c\theta_3 & -s\theta_3 & 0 & a_2 \\ s\theta_3 & c\theta_3 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \\
{}^3_4T &= \begin{bmatrix} c\theta_4 & -s\theta_4 & 0 & a_3 \\ 0 & 0 & -1 & d_4 \\ -s\theta_4 & -c\theta_4 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} {}^4_5T = \begin{bmatrix} c\theta_5 & -s\theta_5 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ s\theta_5 & c\theta_5 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} {}^5_6T = \begin{bmatrix} c\theta_6 & -s\theta_6 & 0 & 0 \\ 0 & 0 & 1 & d_6 \\ -s\theta_6 & -c\theta_6 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (2)
\end{aligned}$$

By multiplying the six matrices in Eq. 2 by $T_6^0 = T_1T_2T_3T_4T_5T_6$ according to law, the relationship between the end of the industrial robot and the base can be obtained, which is the kinematics equation of Fanuc Mate 200iD-4S robot, as shown in Eq. 3.

$$T_6^0 = \prod_{i=1}^6 (T_i^{i-1}(\theta_i)) = \begin{bmatrix} n_x & o_x & a_x & p_x \\ n_y & o_y & a_y & p_y \\ n_z & o_z & a_z & p_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (3)$$

Eq. 3 can solve the forward kinematics and trajectory planning of Fanuc Mate 200iD-4S robot. In order to verify the correctness of the kinematic equation, assuming that the initial conditions of each joint is $\theta_1 = \theta_3 = \theta_4 = \theta_5 = \theta_6 = 0, \theta_2 = 90^\circ$, the value of the end position transformation matrix T_6^0 can be calculated by substituting the initial conditions of each joint into the kinematic equation of Eq.3, and the calculation result is Eq. 4.

$$T_6^0 = \begin{bmatrix} 0 & 0 & 1 & d_4 + d_6 \\ 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & a_2 + a_3 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (4)$$

The first three elements in the last column of the matrix indicate that the end position of the industrial robot moves $d_4 + d_6$ distance along the x_0 -axis in the base coordinate system, 0 distance along the y_0 -axis and $a_2 + a_3$ distance along the z_0 -axis, which is the same as that in the connecting rod coordinate system shown in Fig. 2. This proves that the kinematics equation is correct.

3. Establishment of Fanuc Mate 200iD-4S Robot Model

The D-H parameters of Fanuc Mate 200iD-4S robot can be obtained. The D-H parameters of Fanuc Mate 200iD-4S robot are shown in Fig. 2.

```
robot =
```

LR Mate 200iD-4S:: 6 axis, RRRRRR, stdDH, slowRNE					
j	theta	d	a	alpha	offset
1	q1	0	0	-1.5708	0
2	q2	0	260	0	0
3	q3	0	20	-1.5708	0
4	q4	290	0	1.5708	0
5	q5	0	0	-1.5708	0
6	q6	70	0	0	0

Figure 2. D-H parameter table of Fanuc Mate 200iD-4S robot

4. Forward and Inverse Kinematics Simulation of Fanuc Mate 200iD-4S robot

4.1. Forward Kinematics Simulation.

The forward kinematics solution of industrial robot is to analyze the position and posture change of the end effector of Fanuc Mate 200iD-4S robot on the premise of determining the parameters of the link and setting the changes of the joint nodes. It is to complete the simulation of the forward kinematics solution of industrial robot. The function of forward kinematics simulation in Robotics Toolbox module is `fkine` function, which calls the statement `TP = fkine (robot, theta)`. The robot of this statement is the name of the model of connecting rod industrial robot, `theta` is the angle value of each joint in a certain state, `TP` is the solution of forward kinematics defined by `theta`, and the angle of each joint is the position matrix of the end of `theta`. Taking the pose Figure 3 of the initial state as an example, suppose `theta = [0, pi/2, 0, 0, 0, 0]`, then the homogeneous transformation matrix corresponding to the end can be obtained by `TP = fkine (robot, theta)`. The `TP` result is shown in Fig. 3.

```
>> TP=fkine(robot, theta)
```

```
TP =
```

0	0	1	360
0	-1	0	0
1	0	0	280
0	0	0	1

Figure 3. Posture matrix in `theta` state

The rotation matrix is transformed to $T = \begin{bmatrix} 0 & 0 & 1 \\ 0 & -1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$, which shows that the terminal position

and attitude rotate 90 degrees along Y axis and 180 degrees along Z axis relative to the base coordinate. It can be verified by the rotation homogeneous coordinate transformation formula $R = \text{roty}(\pi/2) * \text{rotz}(\pi)$. In the last column of the matrix, $P_x = d_4 + d_6 = 360\text{mm}$, $P_z = a_2 + a_3 = 280\text{mm}$ are consistent with the pose matrix shown in Formula 3, which further verifies the rationality of the design.

The research of forward kinematics mainly uses coordinate transformation principle to get kinematics equation of industrial robot. Given the value of joint variables, the value of each element in the pose matrix can be calculated, that is, given the value of a group of joint variables. There are and only one calculation result of pose matrix, so the solution of forward kinematics equation of

industrial robot is unique.

4.2. Inverse Kinematics Simulation.

Inverse kinematics is based on the desired pose of the end-effector to calculate the relative joint angles of the industrial robot. Contrary to the solution of positive kinematics, the function of inverse kinematics simulation in Robotics Toolbox module is `ikine` function. Its calling statement is `Q = ikine(robot, TP)`. The robot of this statement is the name of the model of link industrial robot created, TP is an end pose matrix, Q is the solution of inverse kinematics defined by TP, and the angles of each joint angle are obtained. We can take the forward kinematics solution TP as an

$$\text{example. } TP = \begin{bmatrix} 0 & 0 & 1 & d_4 + d_6 \\ 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & a_2 + a_3 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 360 \\ 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 280 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \text{Then each joint of Fanuc Mate 200iD-4S}$$

robot can be calculated by `Q = ikine(robot, TP)`, and the result is shown in Fig. 4.

```
>> Q=ikine(robot, TP)
```

Q =

```
-0.0000 -1.5708 -0.0000 0.0000 0.0000 -0.0000
```

Figure 4. Angles of each joint in TP state

If we only know the homogeneous transformation matrix TP, we need to find the inverse solution of the joint variables, i.e. the motion equation, and get the joint variables of each joint as shown in Fig. 8. The values of the joint variables are the same as those of the previously set joint variables Q rotation angle variables, i.e. the results of the forward kinematics are consistent.

5. Trajectory Planning of Fanuc Mate 200iD-4S Robot

This paper will adopt TPT motion mode, Approach point-to-point motion, set the simulation time of the robot to be 15 s, the initial joint angle of the robot to be $qC=[0,-\pi/2,0,0,0]$, the termination joint angle to be $qM=[\pi/2,0,-\pi/4, \pi/3,0,\pi/4]$, and the time vector of the robot's motion to be $t=[0:0.25:5]$.

Calling statement `Plot(t, q(:, 1))` can get the change curve of each joint angle value, where Q represents displacement and $i=(1, 2, 3, 4, 5, 6)$ represents six joint nodes of the robot respectively. The angular displacement and time distribution curves of six joint nodes of LR Mate 200iD-4S robot are shown in Fig. 5.

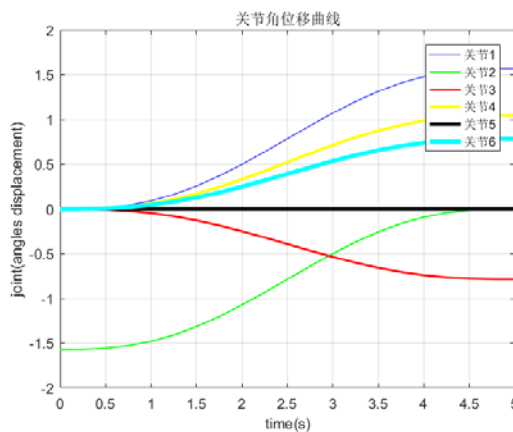


Figure 5. Angular displacement curves of joint nodes of Fanuc Mate 200iD-4S robot

From the motion simulation curve Fig. 5, it can be seen that each joint moves independently and

continuously during the movement process, the angle changes continuously, and there is no sudden change in motion. In order to ensure the stability of the motion, the velocity of the initial point and the end point is set as follows. Through the above-mentioned motion path, the normal condition of each joint in the motion process can be measured. The initial position and posture of smooth motion and angular displacement are 0, the curve of joint 2 is from $-\pi/2$ to 0, the curve of joint 5 is from 0 to 0, and the curve of other joints shows a very smooth trend without discontinuity and sudden change. It can be proved that the structure design of the industrial robot is reasonable and can achieve the desired motion.

Similarly, the velocity and time distribution curves of six joint nodes can be plotted. Its call statement is `plot(t, qd(:, 1))`, in which $i = (1, 2, 3, 4, 5, 6)$. The angular velocity curves of six joints from the starting point to the end point are shown in Fig. 6.

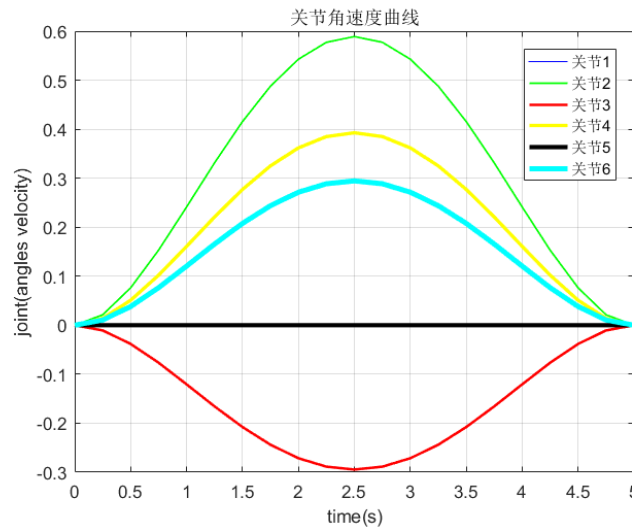


Figure 6. Angular velocity curves of Fanuc Mate 200iD-4S robot joints

From the Fig. 6 of the motion simulation, it can be seen that the angular velocity of each joint increases first and then decreases, the change is continuous and there is no sudden change, so the trajectory motion is more stable.

Similarly, six curves of acceleration angles corresponding to time can be plotted. The corresponding statement called is `plot(t, qdd(:, 1))`, and the $i = (1, 2, 3, 4, 5, 6)$ in that statement. The angular acceleration curves of six joints are shown in Fig. 7.

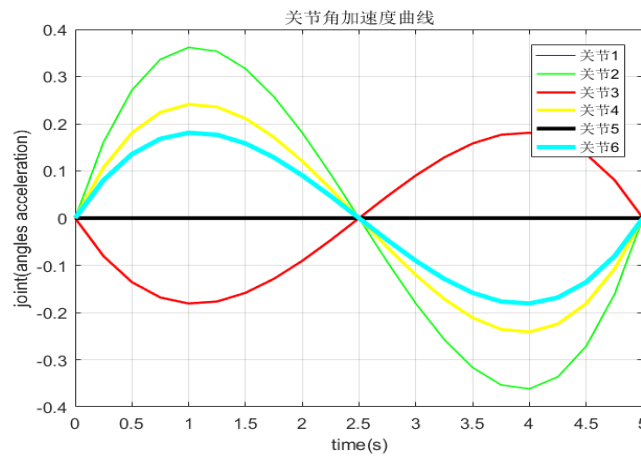


Figure 7. Angular acceleration curves of Fanuc Mate 200iD-4S robot joints

Fig. 7 shows that the angular velocity of each joint in 0-2.5s increases first and then decreases. In 2.5s-5s, the angular velocity of each joint decreases first and then decreases. The process of change is continuous and there is no sudden change. The results show that the robot can accomplish the expected tasks without discontinuous velocity and acceleration. The rationality of connecting rod

parameter design is also explained.

6. Conclusion

(1) The simulation results of forward and inverse kinematics are $TP=Q$. It is proved that the parameter design of Fanuc Mate 200iD-4S robot and the simulation model built in Robotics Toolbox module are correct.

(2) The trajectory planning of Fanuc Mate 200iD-4S industrial robot in joint coordinate space is carried out at any two points (PTP). The motion performance curve shows a smooth trend in the simulation process. There is no discontinuity and sudden change, and there is no vibration. The motion performance of each joint is good.

(3) The trajectory planning of Fanuc Mate 200iD-4S robot is carried out from the joint coordinate space, and the corresponding distribution curves of angular displacement, angular velocity and angular acceleration of each joint node with time are obtained, which verifies the rationality of the parameters of the Fanuc Mate 200iD-4S robot.

(4) In the Matlab software environment, the motion simulation and trajectory simulation of Fanuc Mate 200iD-4S robot are carried out by using Robotics Toolbox module. The simulation results fully meet the requirements of the motion performance of the robot.

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